



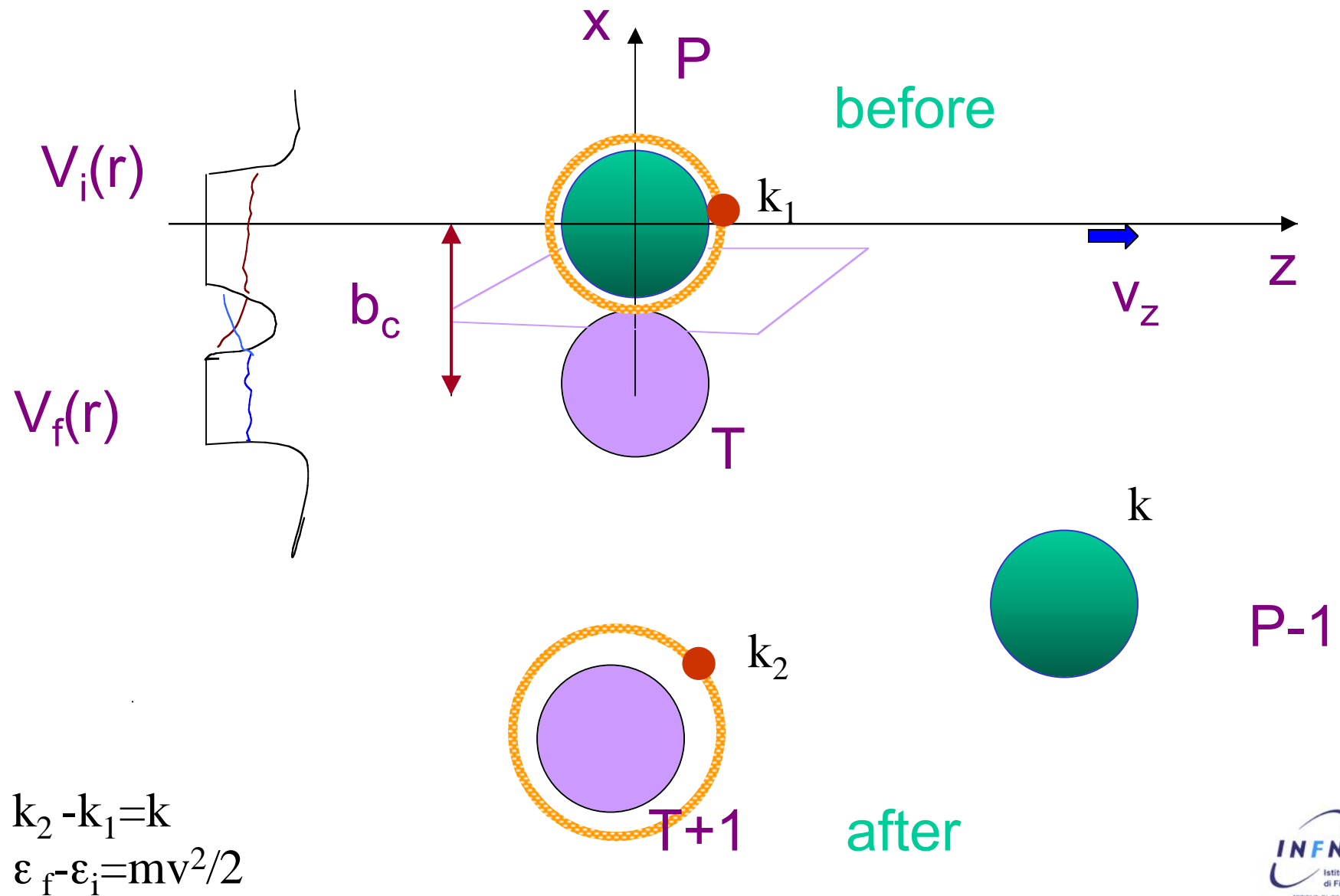
Pisa: Torre Pendente

## Transfer to the continuum method

**D. M. Brink, G. Blanchon, F. Carstoiu**

- (n) or (p) Transfer reactions to unbound states  $\longrightarrow$  breakup  
 $\longrightarrow$  surrogate to free neutron-T scattering  $P+T \longrightarrow (P-n) + (T+n)^*$   
 $\longrightarrow (P-n) + T+n$
- Final state interaction theory (Fermi, Watson...)
- Examples :  
 low-lying resonances in  $^{209}\text{Pb}$   
 the problem of  $^{10}\text{Li}$  ground state  $\longrightarrow ^{11}\text{Li}$
- Optical potentials for weakly bound projectiles

## Transfer to the continuum dynamics



# The neutron Schrödinger equation

$$i\hbar \frac{\partial \Phi}{\partial t} = (T + V_1(r_1, t) + V_2(r_2, t) + V_C(r_1, R(t)))\Phi(t)$$

$$A_{12} = \langle \Phi_{2\,out}(t_2) | G_2 V_2 G_1 | \Phi_{1\,in}(t_1) \rangle$$

$$A_{11} = \langle \Phi_{2\,out}(t_2) | G_1 V_2 G_1 | \Phi_{1\,in}(t_1) \rangle$$

$$A_{01} = \langle \Phi_{2\,out}(t_2) | G_0 V_2 G_1 | \Phi_{1\,in}(t_1) \rangle$$

Semiclassical treatment of core-target relative motion,

**BUT** full QM treatment of n-target interaction

AB and DM Brink, PRC38, 1776 (1988), PRC43, 299 (1991), PRC44, 1559 (1991).

$$\frac{d\sigma}{d\varepsilon_f} = C^2 S \int_0^\infty d\mathbf{b}_c \frac{dP(b_c)}{d\varepsilon_f} P_{el}(b_c), \quad \text{where} \quad P_{el}(b_c) = |S_{cT}|^2$$

$$A_{if} = \frac{1}{i\hbar} \int dt \langle \psi_f(t) | V(r) | \psi_i(t) \rangle$$

$$V(r) = U(r) + iW(r)$$

Same physical content of DWBA when the semiclassical limit applies:

A study of semi-classical approximations for heavy ion transfer reactions,

H. Hasan and D.M. Brink, J Phys G4, 1573 (1978).

Perturbation approach to nucleon transfer in heavy ion reactions,

L. Lo Monaco and D.M. Brink, J.Phys. G, 935 (1985).

$$\psi_i(r, t) = \phi_i(r) e^{-\frac{i}{\hbar} \varepsilon_i t}$$

$$\psi_f^*(r, t) = \phi_f^*(r) e^{\frac{i}{\hbar} \varepsilon_f t}$$

$$\phi_i(r) = -C_i \gamma_i h_{l_i}^{(+)}(\gamma_i r) Y_{l_i m_i}(\theta, \phi)$$

$$\phi_i(r) = C_f k_f \frac{i}{2} [h_{l_f}^{(+)}(k_f r) - S_{l_f}^* h_{l_f}^{(-)}(k_f r)] Y_{l_f m_f}(\theta, \phi)$$

$$A_{if}(\mathbf{k}_f, b_c) \approx \int dk_y \sqrt{k_y^2 + \eta^2} \bar{\phi}_i(d_1, k_y, k_1) \bar{\phi}_f^*(d_2, k_y, k_2)$$

standing the transfer and breakup mechanisms and the best matching ions

$$\begin{aligned}
 \frac{dP_t(b_c)}{d\varepsilon_f} &= \frac{1}{8\pi^3} \frac{mk_f}{\hbar^2} \frac{1}{2l_i + 1} \sum_{m_i} |A_{fi}|^2 \\
 &\approx \frac{4\pi}{2k_f^2} \sum_{j_f} (2j_f + 1) (|1 - \bar{S}_{j_f}|^2 + 1 - |\bar{S}_{j_f}|^2) (1 + F_{l \rightarrow j}) B_{l_f, i} \\
 &= \sigma_{nN}(\varepsilon_f) \mathcal{F},
 \end{aligned}$$

elastic      absorption

enhancement factor of of final state  
interaction theory

$$B_{l_f, l_i} = \frac{1}{4\pi} \left[ \frac{k_f}{mv^2} \right] |C_i|^2 \frac{e^{-2\eta b_c}}{2\eta b_c} M_{l_f l_i}$$

k<sub>f</sub> ≡ (iη, k<sub>z</sub>)
|ψ̃<sub>i</sub>|<sup>2</sup> Fourier transform of initial w. f.
angular parts of ψ<sub>i, f</sub>

$$k_1 = -\frac{\varepsilon_i - \varepsilon_f + \frac{1}{2}mv^2}{\hbar v}$$

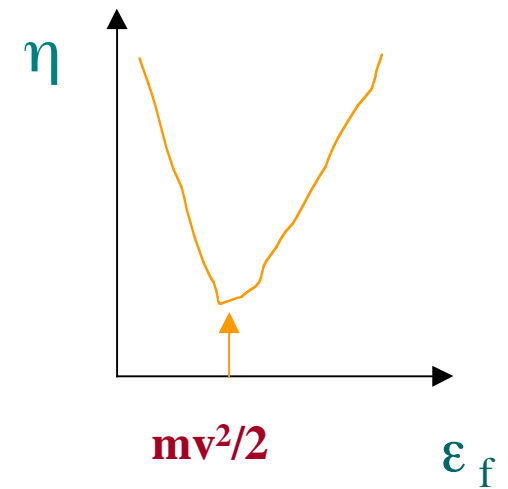
$$k_2 = -\frac{\varepsilon_i - \varepsilon_f - \frac{1}{2}mv^2}{\hbar v}.$$

$$\eta^2 = \gamma_i^2 + k_1^2 = k_2^2 - k_f^2$$

$$\gamma_i^2 = -\frac{2m\varepsilon_i}{\hbar^2}$$

$$k_f^2 = \frac{2m\varepsilon_f}{\hbar^2}$$

$$\mathbf{k}_f \equiv (\mathbf{k}_\perp, k_z) = (i\eta, k_2)$$



If both initial and final state have  $l=0$

Bound to bound

$$\sigma(\varepsilon_f) = \frac{\pi}{2} |C_i C_f|^2 \left[ \frac{\hbar}{mv} \right]^2 \int_0^\infty d\mathbf{b}_c \frac{e^{-2\eta b_c}}{\eta b_c} e^{(-\ln 2 \exp[(R_s - b_c)/a])}$$

Bound to continuum

$$\frac{d\sigma}{d\varepsilon_f} = \left( \frac{\sin \delta_0}{k_f} \right)^2 |C_i|^2 \frac{mk_f}{\hbar^2} \left[ \frac{\hbar}{mv} \right]^2 \int_0^\infty d\mathbf{b}_c \frac{e^{-2\eta b_c}}{\eta b_c} e^{(-\ln 2 \exp[(R_s - b_c)/a])}$$

scattering length

$$a_s = - \lim_{k \rightarrow 0} \frac{\tan \delta_0}{k}$$



# Where do we stand?...just an example.....

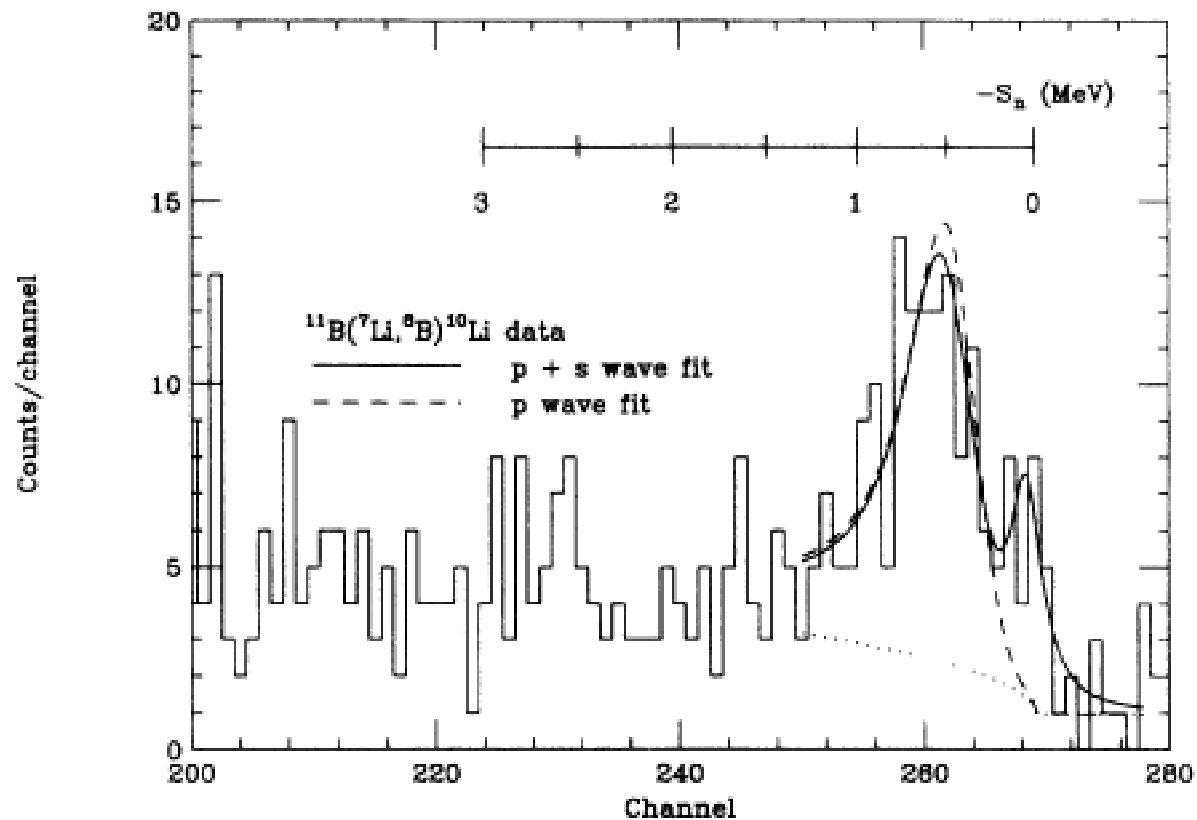
PHYSICAL REVIEW C

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## Low-lying structure of $^{10}\text{Li}$ in the reaction $^{11}\text{B}(^7\text{Li}, ^8\text{B})^{10}\text{Li}$

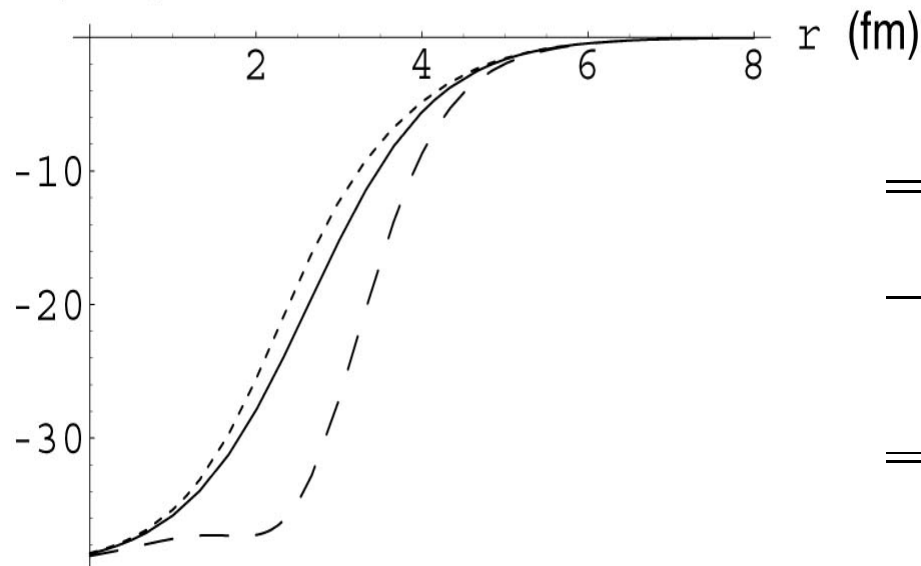
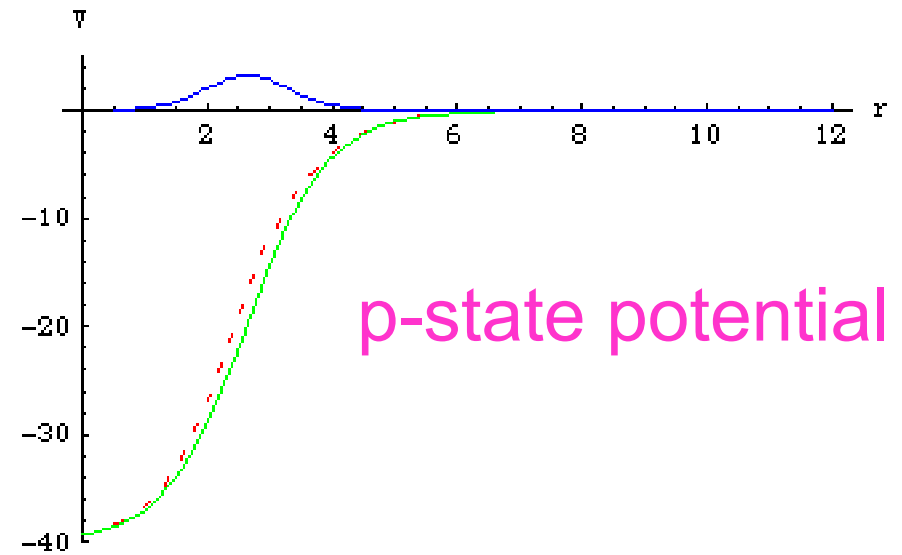
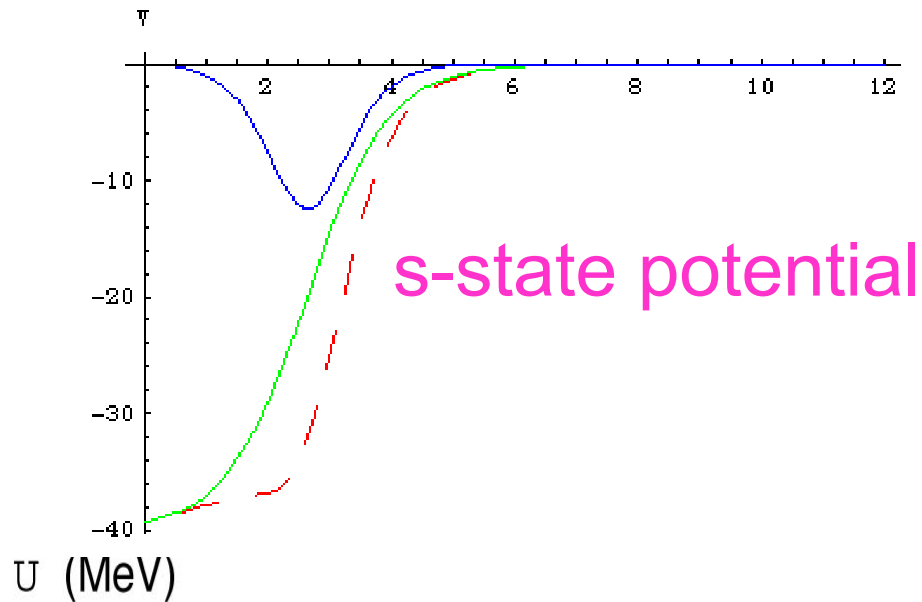
B. M. Young, W. Benenson, J. H. Kelley, N. A. Orr,\* R. Pfaff, B. M. Sherrill, M. Steiner, M. Thoennessen,  
J. S. Winfield, J. A. Winger,<sup>†</sup> S. J. Yennello,<sup>†</sup> and A. Zeller  
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East Lansing, Michigan 48824*  
(Received 22 June 1993)



# Potential model for n+<sup>9</sup>Li continuum

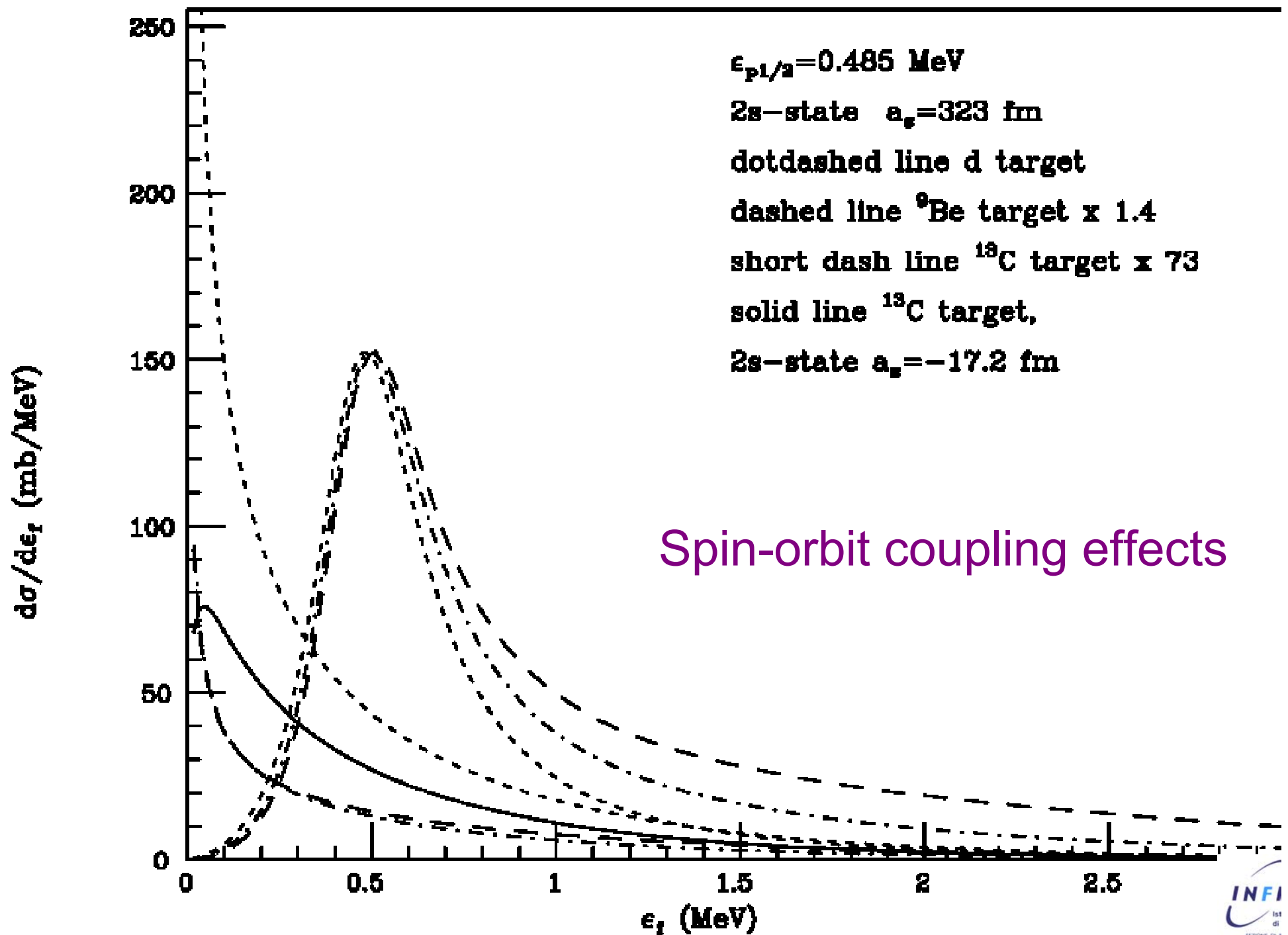
$$U(r) = V_{WS} + \alpha \delta V$$

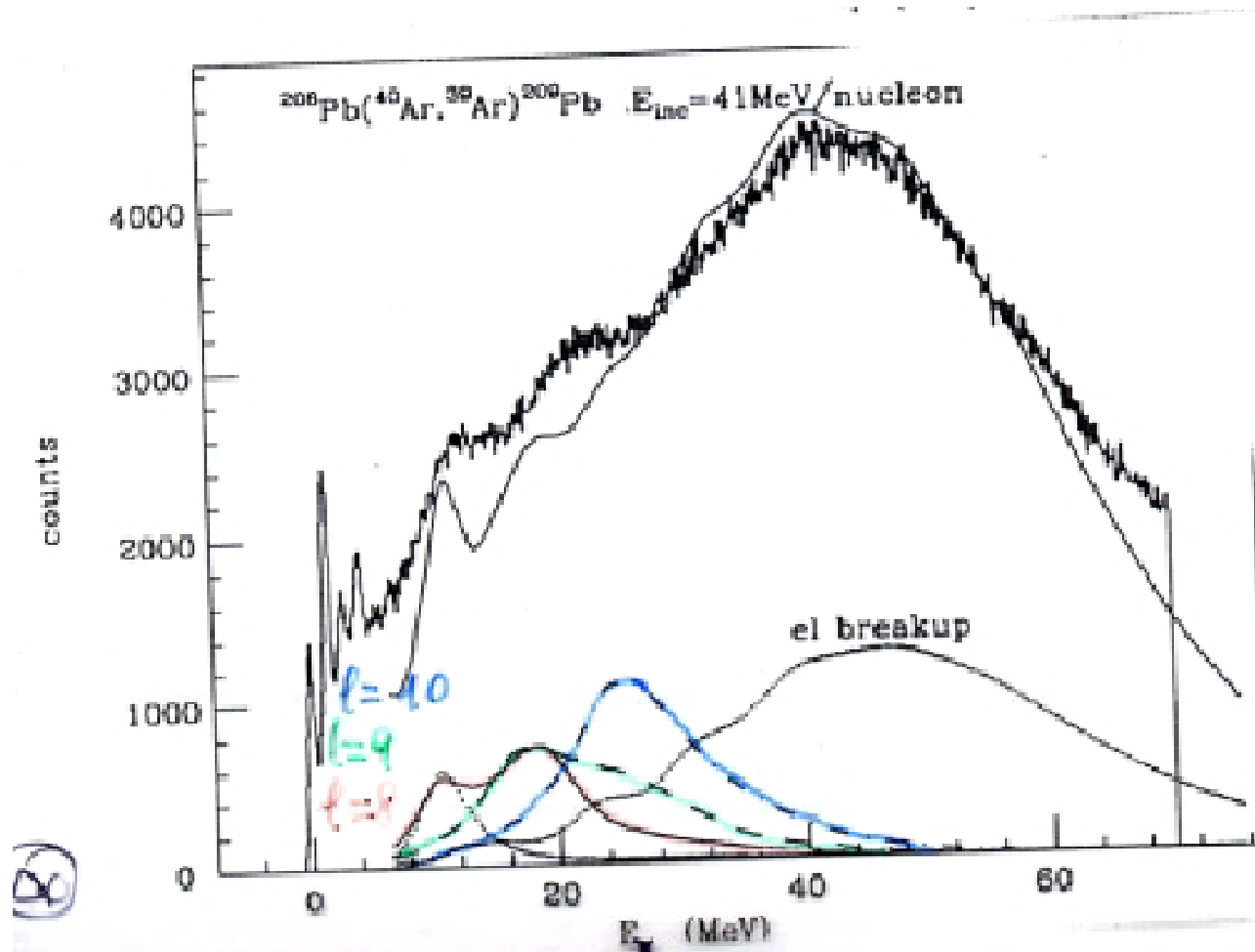
Woods-Saxon+surface-vibration coupling



## Resonance states in <sup>10</sup>Li

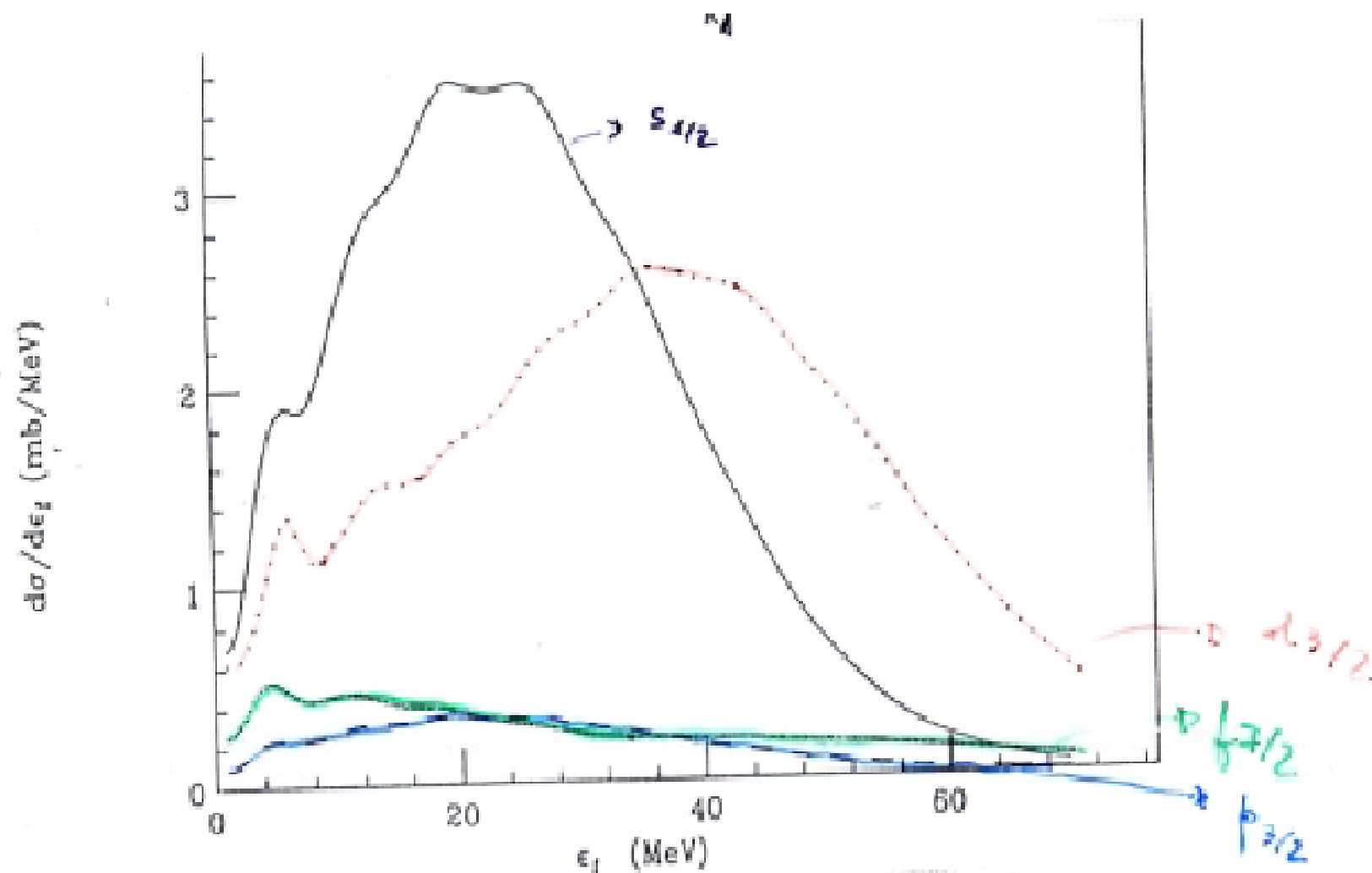
	$\varepsilon_{res}$ (MeV)	$\Gamma$ (MeV)	$a_s$ (fm)	$\alpha$ (MeV)
$2s_{1/2}$			323	-12.5
			-17.20	-10.0
$1p_{1/2}$	0.595	0.48		3.3





Bonaccorso,  
Lentz, Sjöberg  
PR C 49 (1994) 329

n- $^{208}\text{Pb}$  Optical potential from Mahaux and Sartor NP A493 ( 1989) 157



J of initial orbital determined by core momentum distributions.

A. B. and D.M. Brink, PRC44, 1559 (1991); PRC58, 2864 (1998), A.B,PRC60,546046

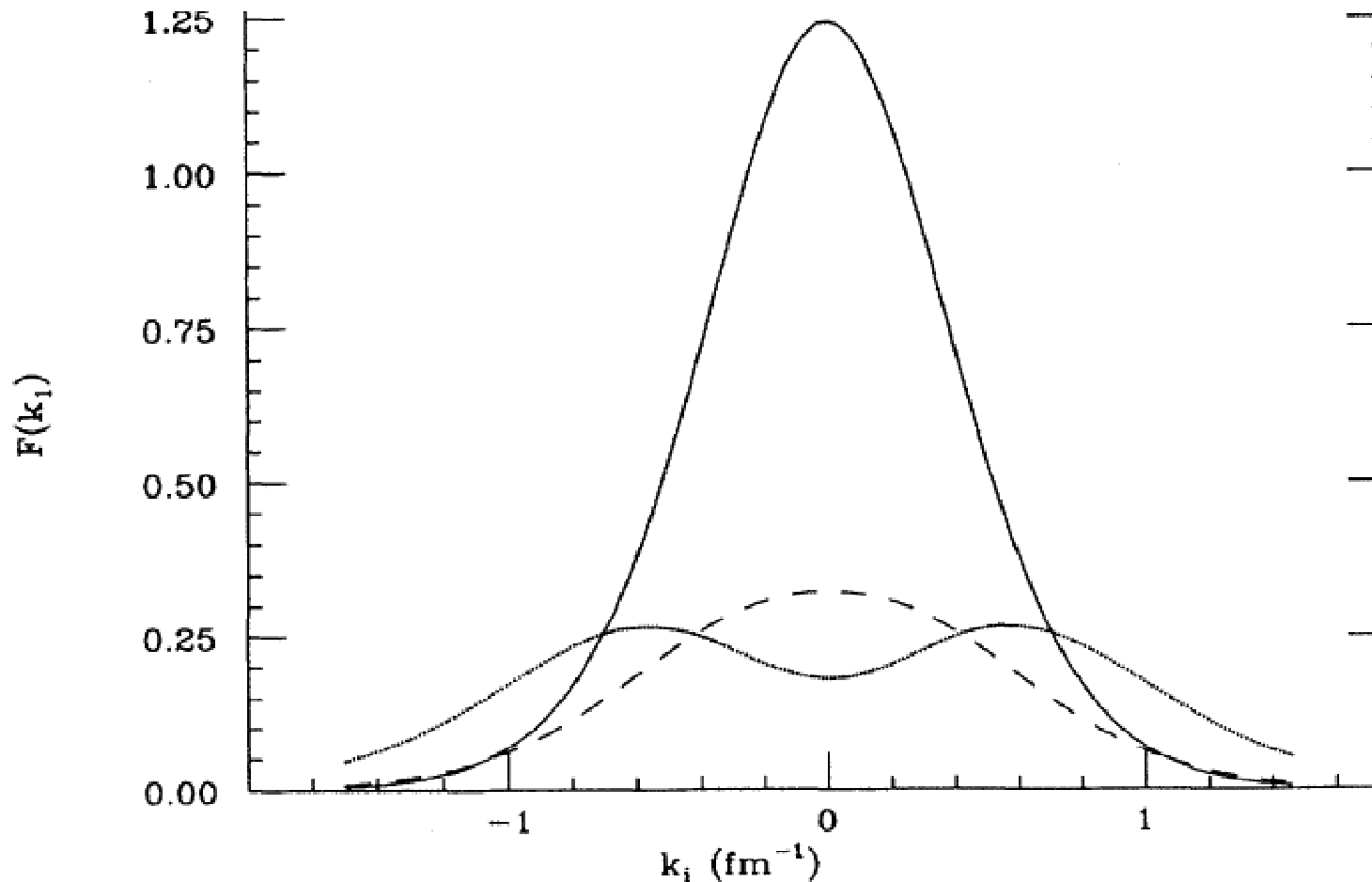


FIG. 11. Initial-state momentum distributions in  $^{20}\text{Ne}$  according to Eq. (2.3a). The solid curve is for the  $2s_{1/2}$  state, the dashed curve is for the  $1p_{1/2}$ , while the dotted curve is for the  $1d_{5/2}$  state.

# Optical potentials of halo and weakly bound nuclei

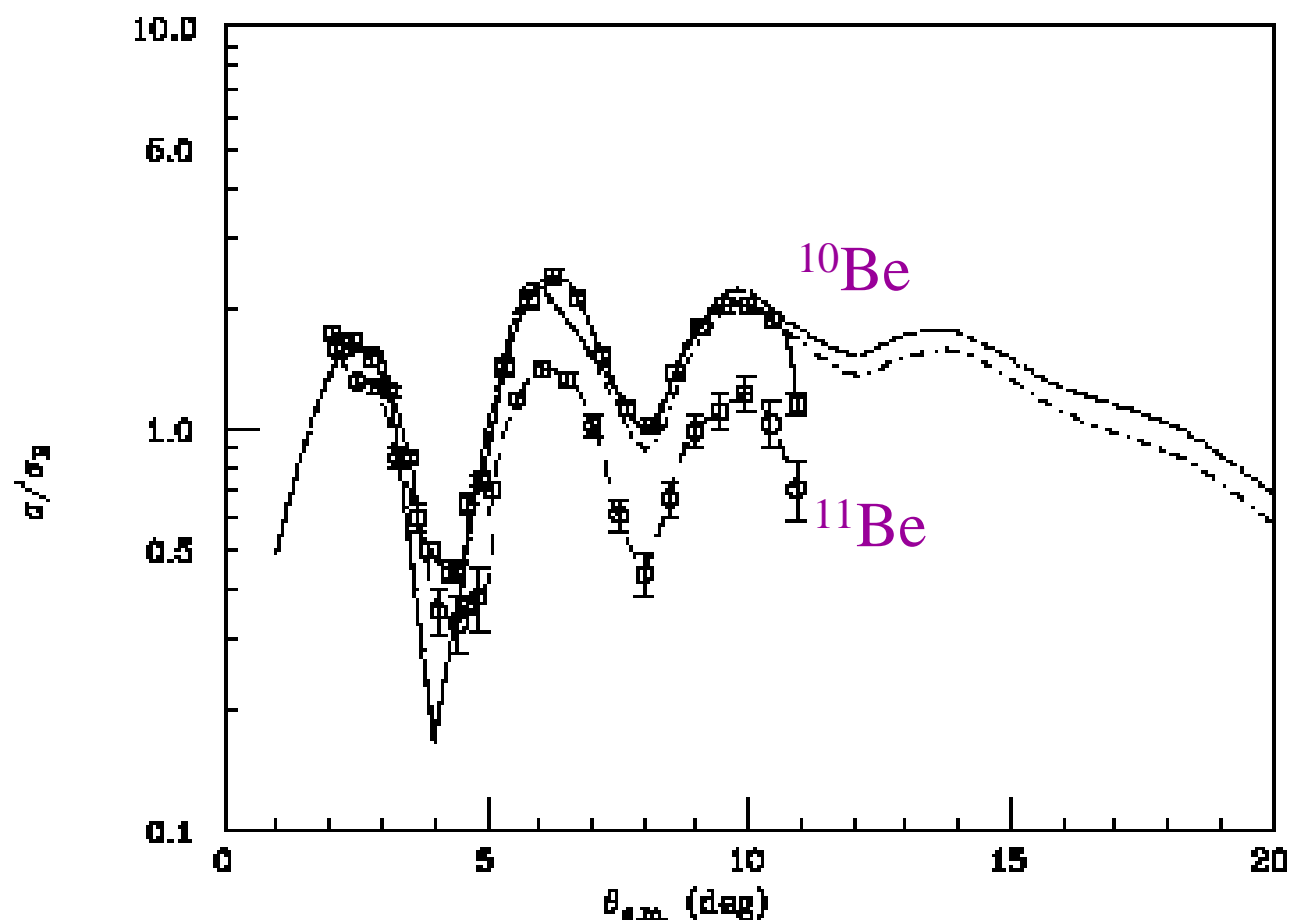
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<sup>b</sup> *Institute of Atomic Physics, PO Box MG-6, Bucharest, Romania*

- [25] R. A. Broglia, and A. Winther, *Heavy Ion Reactions*, Benjamin, Reading, Mass, 1981.
- [26] R. A. Broglia, G. Pollaro and A. Winther, Nucl. Phys. **A361**, 307 (1981).
- [27] A. Bonaccorso, G. Piccolo, D. M. Brink, Nucl. Phys. **A441** (1985) 555.
- [28] Fl. Stancu and D. M. Brink, Phys. Rev. C **32**, 1937 (1985).

Elastic scattering and optical potential description select important reaction channels  
Typical experiment to be done with EXCYT and MAGNEX going to large angles





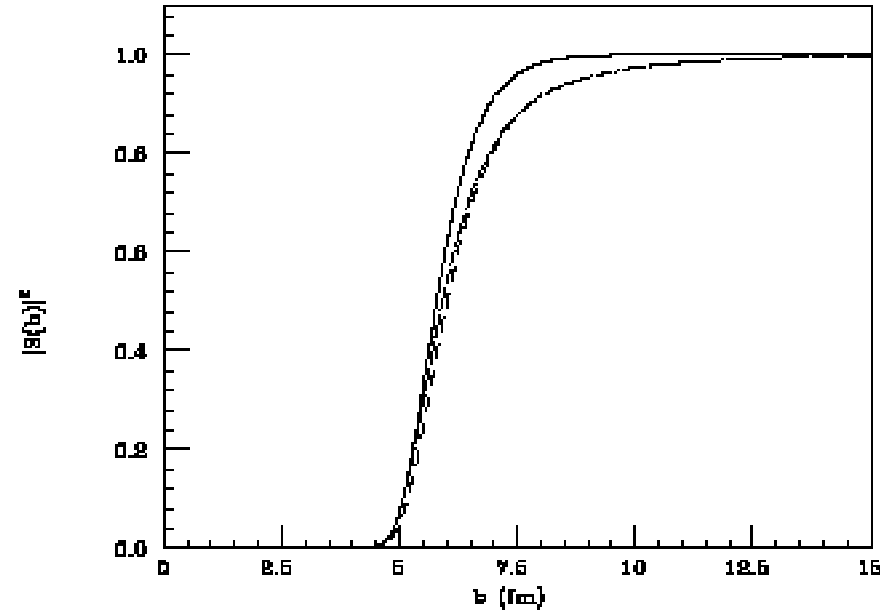


Figure 2: S-matrix values as a function of the impact parameter for the system  $^{11}\text{Be} + ^9\text{Be}$  at 50A.MeV. Solid line is  $|S_{CT}|^2$ , dashed and dotted lines are  $|S_{NN}|^2$  calculated with the two prescriptions for the breakup probability discussed in the text.

## Optical potential from phase shift

$$|S_{NN}(b_c)|^2 = e^{-4\delta_I(b_c)}.$$

$$\delta_I(b_c) = -\frac{1}{2\hbar} \int_{-\infty}^{+\infty} (W_V(\mathbf{r}(t)) + W_S(\mathbf{r}(t))) dt$$

For each impact parameter where

$$\int_{-\infty}^{+\infty} W_S(b_c z) dz = -\frac{\hbar v}{2} p_{b_{up}}(b_c)$$

$$W_S(r) \approx W_0 e^{-\frac{r-R_s}{\alpha}}$$

$$W_0 \equiv W_0(R_s) = -\frac{\hbar v}{2} p_{b_{up}}(R_s) \frac{1}{\sqrt{2\pi\alpha R_s}}$$

$$\begin{aligned}\eta &\approx \gamma \\ \gamma &= \sqrt{2mS_n}/\hbar \\ \alpha &\approx 1/(2\gamma)\end{aligned}$$

$$\begin{aligned}
1 - |S_{NN}(b_c)|^2 &\approx 1 - |S_{CT}(b_c)|^2 e^{-P_{b_{up}}(b_c)} \\
&= 1 - |S_{CT}(b_c)|^2 (1 - P_{b_{up}}(b_c)) \\
&= 1 - |S_{CT}(b_c)|^2 + |S_{CT}(b)|^2 P_{b_{up}}(b_c)
\end{aligned}$$

$$\begin{aligned}
\sigma_{NN} &= 2\pi \int b_c db_c (1 - |S_{NN}(b_c)|^2) \\
&\approx \sigma_{CT} + \sigma_{b_{up}}
\end{aligned}$$